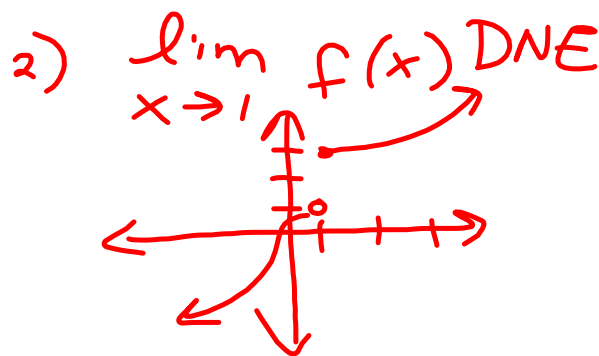
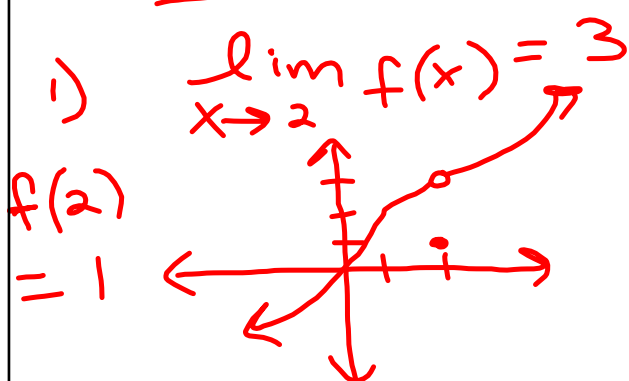


## Warm-up

Find the limit graphically:



Find the limit numerically:

3)  $\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x - 1} = 5$

4)  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

## Analytical limits (1.3)

Try to plug-in 1st

- ① Factor, cancel + plug-in
- ② rationalize using the conjugate (sq. rt.)
- ③ simplify complex fractions
- ④
- ⑤

$$\begin{aligned} \text{ex: } \lim_{x \rightarrow 1} & \frac{x^2 + 3x - 4}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x+4)}{\cancel{x-1}} \\ &= \lim_{x \rightarrow 1} (x+4) \\ &= \boxed{5} \end{aligned}$$

$$\text{ex: } \frac{x^2 + 3x - 4}{x - 1} = \frac{(x + 4)\cancel{(x - 1)}}{\cancel{x - 1}}$$

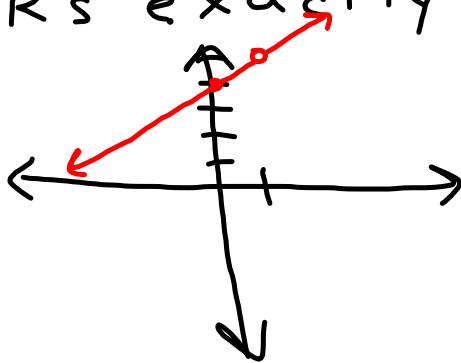
removable discontinuity  
(hole) at  $x = 1$

$$\text{ex: } \frac{x}{x - 1} \quad \text{cannot remove}$$

red flag  
non-removable (asymptote)  
disc. at  $x = 1$ .

$$\lim_{x \rightarrow 1} \frac{(x+4)\cancel{(x-1)}}{\cancel{x-1}}$$
$$= 5$$

hole is at (1, 5)  
looks exactly like  $x+4$



$$\text{ex: } \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$$

$$\lim_{x \rightarrow 2} \frac{\cancel{(x - 2)}(x^2 + 2x + 4)}{\cancel{x - 2}}$$

$$\lim_{x \rightarrow 2} (x^2 + 2x + 4) = \boxed{12}$$

$$\text{ex: } \lim_{x \rightarrow 1} (x^2 + 5) \\ = \boxed{6}$$

If continuous at the  
x-value you will  
always be able to plug-in.

$$\text{ex: } \lim_{x \rightarrow 0} \frac{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)}{x(\sqrt{x+1} + 1)}$$

$$\lim_{x \rightarrow 0} \frac{x+1-1}{x(\sqrt{x+1} + 1)}$$

$$\lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+1} + 1)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+1} + 1}$$

$$= \boxed{\frac{1}{2}}$$



Try

$$\textcircled{1} \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = 10$$

$$\textcircled{2} \lim_{x \rightarrow 2} \frac{x^2 + 5x - 14}{x - 2} = 9$$

$$\textcircled{3} \lim_{x \rightarrow \pi} \sin x = 0$$

$$\textcircled{4} \lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x-4} = \frac{1}{6}$$

$$\text{ex: } \lim_{x \rightarrow 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{\frac{x}{1} \cdot 4(x+4)}$$

$$\lim_{x \rightarrow 0} \frac{4 - (x+4)}{4x(x+4)}$$

$$\lim_{x \rightarrow 0} \frac{4 - x - 4}{4x(x+4)} = \lim_{x \rightarrow 0} \frac{-x}{4x(x+4)}$$

$$\lim_{x \rightarrow 0} \frac{-1}{4(x+4)} = \frac{-1}{16}$$