

$$51) \lim_{x \rightarrow 0} \frac{x}{x^2 - x}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{x}}{\cancel{x}(x-1)}$$

$$\lim_{x \rightarrow 0} \frac{1}{x-1}$$
$$= \boxed{-1}$$

$$59) \quad \text{FL} \quad \lim_{x \rightarrow 0} \frac{(\sqrt{x+5} - \sqrt{5})(\sqrt{x+5} + \sqrt{5})}{x(\sqrt{x+5} + \sqrt{5})}$$

$$\lim_{x \rightarrow 0} \frac{x + \cancel{5} - \cancel{5}}{x(\sqrt{x+5} + \sqrt{5})}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+5} + \sqrt{5})}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{x+5} + \sqrt{5}} = \frac{1 \cdot \sqrt{5}}{2\sqrt{5} \cdot \sqrt{5}} = \boxed{\frac{\sqrt{5}}{10}}$$

$$61) \lim_{x \rightarrow 0} \frac{3 \cancel{(3+x)}^1}{\cancel{3+x}} - \frac{1}{\cancel{3}} \cdot \cancel{3(3+x)}$$

$$x \cdot 3(3+x)$$

$$\lim_{x \rightarrow 0} \frac{3 \overbrace{-(3+x)}}{3x(3+x)}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{3} - \cancel{3} - x}{3x(3+x)}$$

$$\lim_{x \rightarrow 0} \frac{-x}{3x(3+x)} = \lim_{x \rightarrow 0} \frac{-1}{3(3+x)}$$

$$= \sqrt{\frac{-1}{9}}$$

## Analytical limits (1.3) cont'd

ex:  $\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

$$f(x) = x^2$$

$$\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

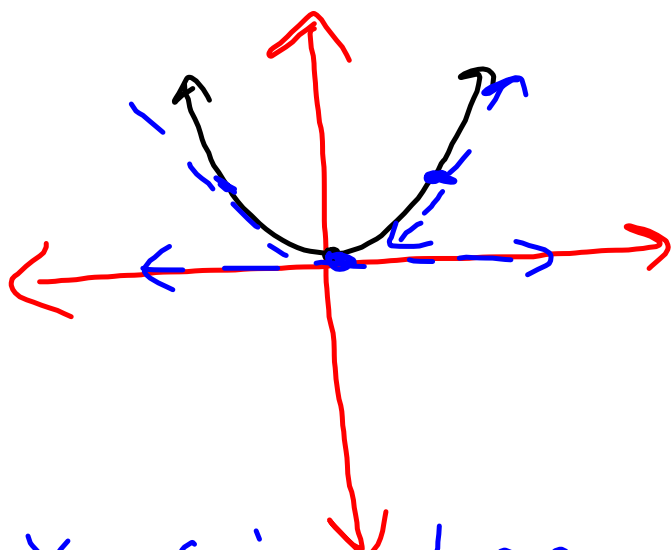
$$\lim_{h \rightarrow 0} \frac{(x + h)^2 - x^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$\lim_{h \rightarrow 0} (2x+h)$$
$$= 2x$$

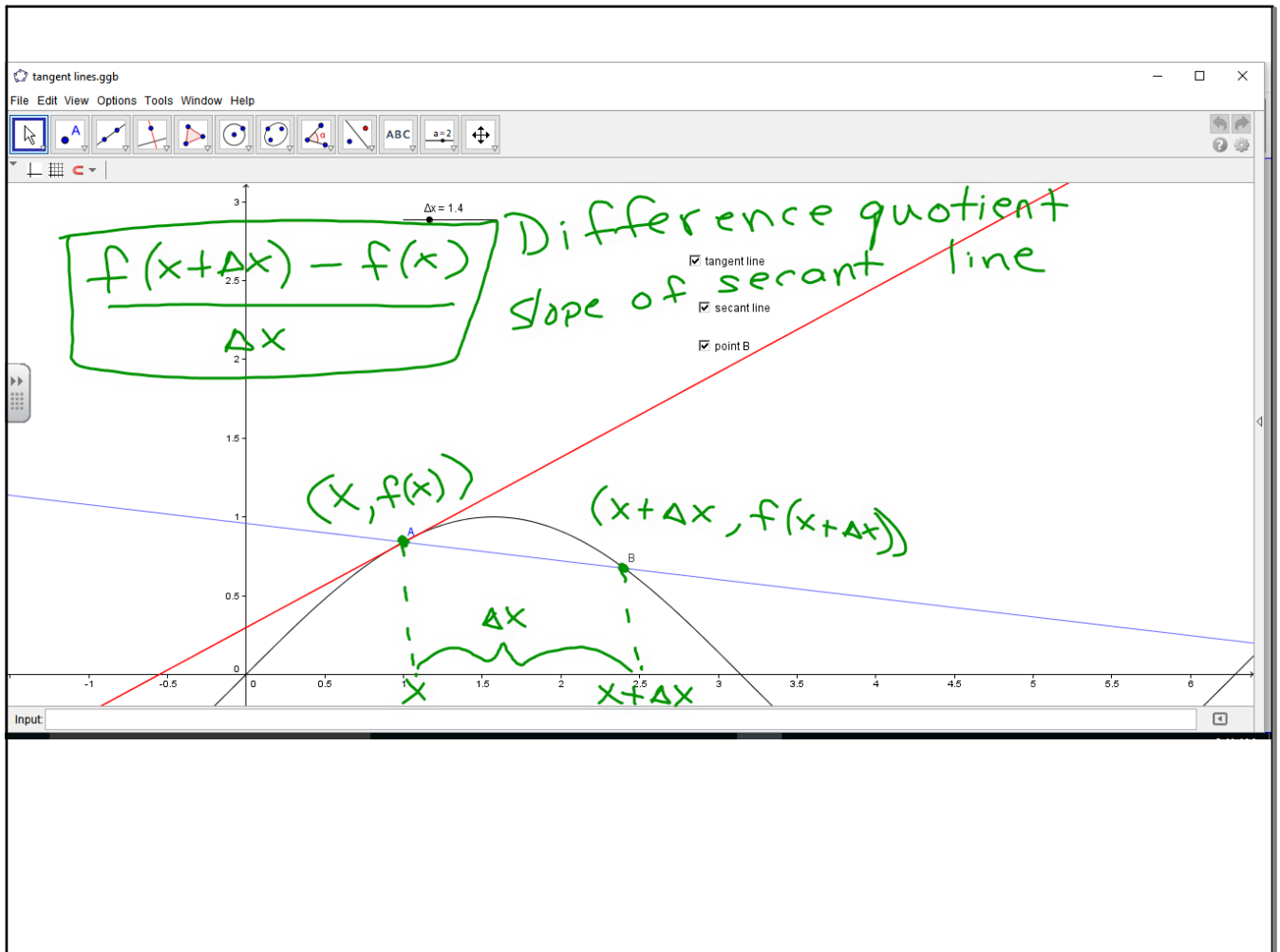
$$f(x) = x^2$$



at  $x=0$ : slope = 0

at  $x=2$ : slope  $2(2) = 4$

at  $x=-2$ : slope  $2(-2) = -4$



$$\text{ex: } \lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$