Warm-up

Find the limit:

1) $\lim\limits_{x \to a} f(x)$
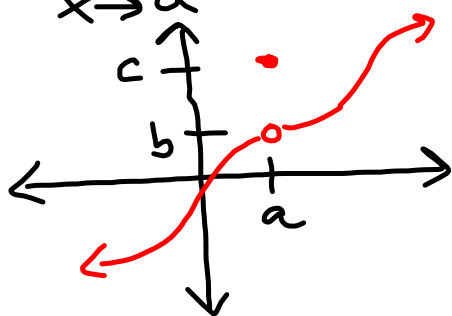
2) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

$\lim\limits_{x \to 2} (x+3)\dfrac{\cancel{(x-2)}}{\cancel{x-2}}$

$= 5$

3) $\lim\limits_{x \to 3} \dfrac{\sqrt{x} - \sqrt{3}}{x - 3}$

4) $\lim\limits_{h \to 0} \dfrac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$

$$\lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - \boxed{(x^2 + 2x)}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$\lim_{h \to 0} \frac{h^2 + 2xh + 2h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(h + 2x + 2)}{\cancel{h}}$$

$$\lim_{h \to 0} (h + 2x + 2) = \boxed{2x + 2}$$

$$3) \lim_{x \to 3} \frac{(\sqrt{x} - \sqrt{3})(\sqrt{x} + \sqrt{3})}{(x - 3)(\sqrt{x} + \sqrt{3})}$$

$$\lim_{x \to 3} \frac{x - 3}{(x - 3)(\sqrt{x} + \sqrt{3})}$$

$$\lim_{x \to 3} \frac{1}{\sqrt{x} + \sqrt{3}}$$

$$= \boxed{\frac{1}{2\sqrt{3}}}$$

$$\underline{\text{Derivative rules}} \ \underline{(\text{ch.2})}$$

① chain rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

ex: $\dfrac{d}{dx}\left(x^2+5\right)^{10} = 10\left(x^2+5\right)^9 \cdot 2x$

$$= 20x\left(x^2+5\right)^9$$

② power rule

$$\frac{d}{dx} x^n = n x^{n-1}$$

ex: $\dfrac{d}{dx} x^5 = 5x^4$

③ constant multiple rule

$\frac{d}{dx} a f(x)$ a is a constant

= $a f'(x)$

ex: $\frac{d}{dx} 5 \sin x = 5 \cos x$

④ product rule

$\frac{d}{dx} f(x) \cdot g(x) = f'(x) g(x) + f(x) \cdot g'(x)$

ex: $f(x) = 5x \sin x$

= $5 \sin x + 5x \cos x$

⑤ quotient: $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x) g(x) - f(x) g'(x)}{[g(x)]^2}$

ex: $\frac{d}{dx} \frac{5x}{x^2+3} = \frac{5(x^2+3) - 5x \cdot 2x}{(x^2+3)^2}$

= $\frac{5x^2 + 15 - 10x^2}{(x^2+3)^2} = \frac{-5x^2 + 15}{(x^2+3)^2}$